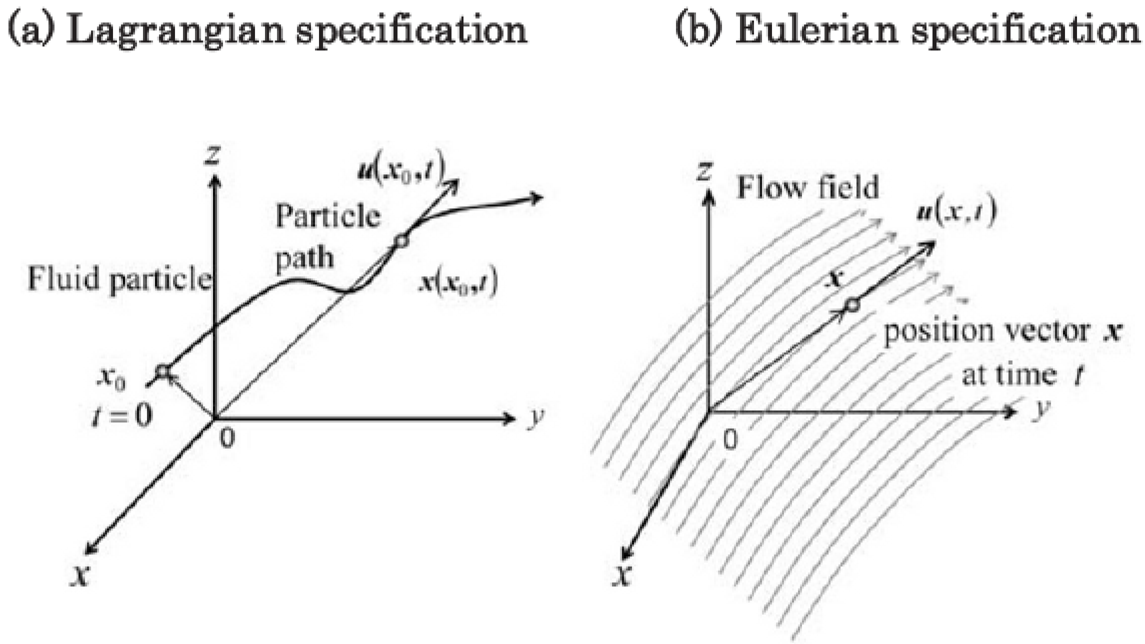
Motion of a Fluid



Description of fluid motion

- The motion of a fluid can be determined, when the velocity at every point of the space is occupied by fluid motion.
- Therefore, to express the velocity with independent variables, there are two distinct methods available,
  - (i) Lagrangian method ; (ii) Eulerian method.

Motion of a Fluid

- Lagrangian method
- traces the motion of a fluid particle in space with time
- The position vector $\boldsymbol{x}$ of the fluid particle, considering a motion relative to a given frame of reference at time *t* , can be expressed as

$$\boldsymbol{x} = \boldsymbol{x}(\boldsymbol{x}_0, t)$$

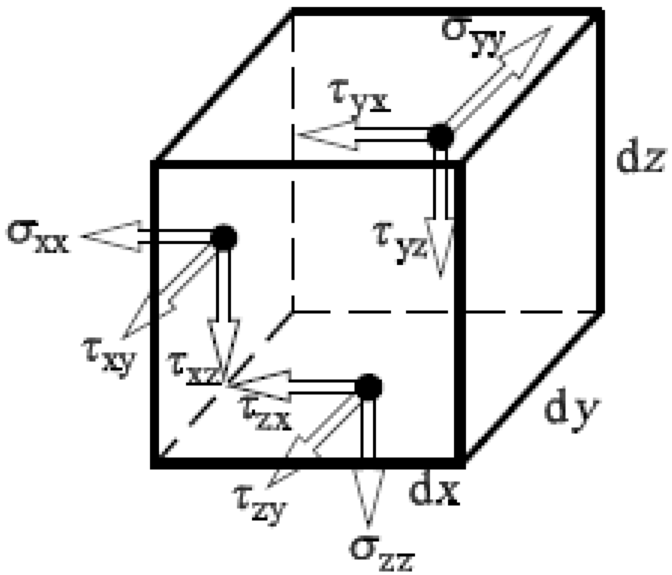
- where ***x***0 and *t* are independent parameters, and ***x***0 is the original position at *t* = 0
- The velocity ***u*** and acceleration ***a*** at time *t*
- can be written similarly as

(ii) Eulerian method.

$$\boldsymbol{u} = \boldsymbol{u}(\boldsymbol{x}_0, t) = \frac{\partial \boldsymbol{x}}{\partial t} \qquad \boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}_0, t) = \frac{\partial \boldsymbol{u}}{\partial t}$$

Stresses in Fluid: Viscous stress

- The *general theory of liquid friction* tells us that when the shape of a single liquid element is changed, stresses arise that are similar to those of elastic bodies.
- The difference lies in the fact that these stresses are not proportional to the changes of shape, but rather to the velocities of the changes of shape.
- The equations for the nine stress components (three each on the three surfaces perpendicular to the coordinate axes (see the Figure)) therefore read (for Newtonian and incompressible fluid)



Normal and shear stress at volume element dV = dx·dy·dz

$$\sigma_{xx} = 2 \cdot \mu \cdot \frac{\partial u}{\partial x}, \qquad \tau_{xy} = \tau_{yx} = \mu \cdot \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right),$$

$$\sigma_{yy} = 2 \cdot \mu \cdot \frac{\partial v}{\partial y}, \qquad \tau_{yz} = \tau_{zy} = \mu \cdot \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right),$$

$$\sigma_{zz} = 2 \cdot \mu \cdot \frac{\partial w}{\partial z}, \qquad \tau_{zx} = \tau_{xz} = \mu \cdot \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right).$$

Stresses in Fluid: Viscous stress

- For *general Newtonian fluids.*

$$\sigma_{xx} = 2 \cdot \mu \cdot \frac{\partial u}{\partial x} - \frac{2}{3} \cdot \mu \cdot \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right),$$

$$\sigma_{yy} = 2 \cdot \mu \cdot \frac{\partial v}{\partial y} - \frac{2}{3} \cdot \mu \cdot \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right),$$

$$\sigma_{zz} = 2 \cdot \mu \cdot \frac{\partial w}{\partial z} - \frac{2}{3} \cdot \mu \cdot \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right),$$

$$\tau_{yx} = \tau_{xy} = \mu \cdot \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right),$$

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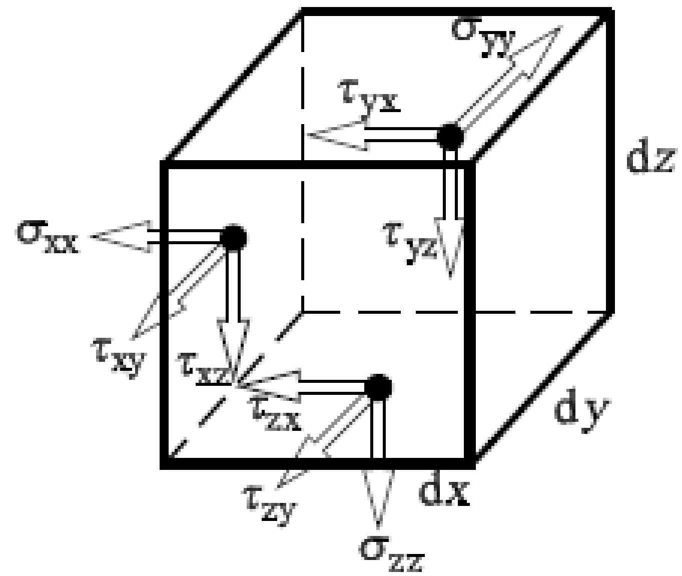


Normal and shear stress at volume element dV = dx·dy·dz

Continuity Equation

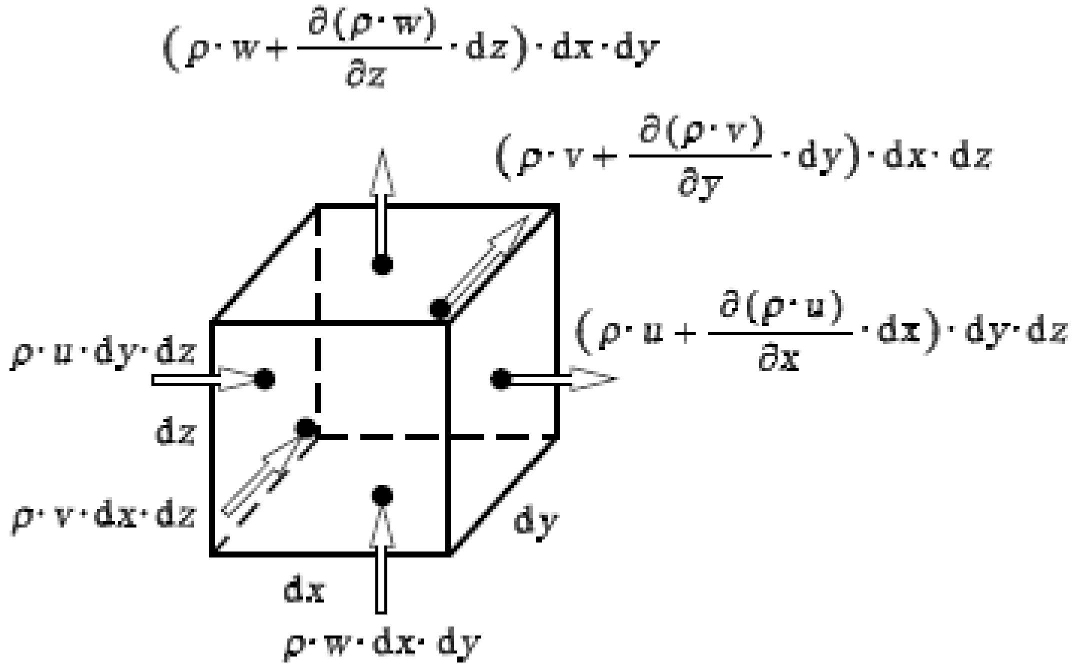
- In general, the conservation of mass at a volume element may be formulated as follows:

The rate of change of mass in a volume element

= $\sum$ the mass fluxes into the volume element

− $\sum$ the mass fluxes out of the volume element.

Continuity Equation



Mass fluxes entering and exiting the volume element dV