Department of Mechanical Engineering, BUET.

ME 6125: Mechanics of Viscous Fluid

Assignment-5: *Boundary layer flow*

(**Due Date: 17 August 2013**, **Sunday**. *Submit hard-copy, at class*)

1. Figure 1 illustrates a two-dimensional jet entering a reservoir that contains a stationary fluid. A solution is sought to the laminar boundary layer equations for this situation. Assuming that there is no pressure gradient along the jet, look for a similarity solution for the stream function of the following form: $\psi(x,y) = 6\alpha\nu x^{1/3} f(\eta)$ where $\eta = \alpha \dfrac{y}{x^{2/3}}$

In the expressions above, α is a dimensional constant and ν is the kinematic viscosity of the fluid. Obtain an expression for the function *f(η)* in this solution and the boundary conditions that it has to satisfy. From the solution for *f(η)*, obtain the solution for the streamfunction ψ (x,y).

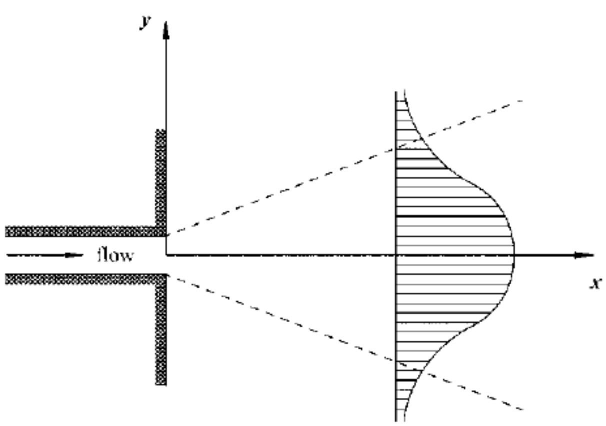

Figure 1. Jet entering a reservoir.

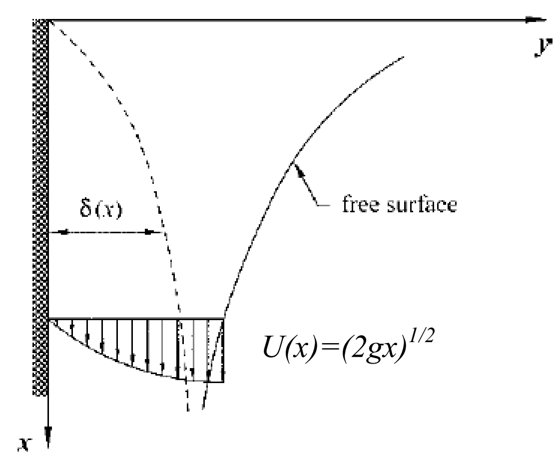

Figure 2. Liquid flowing down a vertical surface.

2. Figure 2 shows a viscous, incompressible liquid flowing down a vertical surface. A boundary layer develops on the vertical surface and grows to approach the free surface. Taking into account the force due to gravity, write down the boundary-layer equations for this flow configuration. From these equations obtain the corresponding momentum integral. Hence, by employing a second-order polynomial for the velocity distribution, obtain an expression for the boundary-layer thickness $\delta(x)$.

3. The solution to the boundary layer equations corresponding to flow in a convergent channel resulted in the following ordinary differential equation:

$$f''' + 1 - (f')^2 = 0$$

Show that this third-order, nonlinear, ordinary differential equation may be integrated to give

$$f'(\eta) = 3\tanh^2\left[\frac{\eta}{\sqrt{2}} + 1.146\right] - 2$$

where the primes denote differentiation with respect to η.
