Department of Mechanical Engineering, BUET.
ME 6125: Mechanics of Viscous Fluid

Assignment-4: *Low Reynolds number flow*

(**Due Date: 17 August 2013**, **Sunday**. *Submit hard-copy, at class*)

1. Consider the low Reynolds number flow of a fluid between two parallel flat plates which are fixed at a small distance *h* apart (Hele-Shaw flow). After performing the leading order approximation, show that

$$u_x = -\frac{1}{2\nu}\frac{\partial p}{\partial x} z(h-z) \qquad u_y = -\frac{1}{2\nu}\frac{\partial p}{\partial y} z(h-z)$$

where, $\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0$, $\frac{\partial p}{\partial z} = 0$, and Boundary conditions: $z = 0, h: u_x, u_y, u_z = 0$.

This leads to the mean velocity in the plane of the Hele-Shaw cell given by,

$$\overline{u}_x = \int_0^h u_x dz = -\frac{h^2}{12\nu}\frac{\partial p}{\partial x}, \qquad \overline{u}_y = \int_0^h u_y dz = -\frac{h^2}{12\nu}\frac{\partial p}{\partial y}$$

which predicts that the mean velocity field for a Hele-Shaw flow corresponds to a potential flow in the two dimensions with the "mean" velocity potential given by $-h^3p/12\nu$ satisfying the Laplace equation!

2.Consider a sphere of radius $r = a$ rotating with angular velocity ω about a diameter so that $Re=\omega a/\nu<<1$. Use the symmetries in the problem to solve the mass and momentum equations directly for the azimuthal velocity $u_\phi (r,\theta)$. Then find the shear stress and torque on the sphere.

3. Consider the fluid flow between two plates (Plane lubrication film flow: basic flow of tribology) as shown in Figure 1. In the figure, U_P = velocity of the moving lower plate; α = inclination angle between the plates; $R(x_1)$ = variation of plate distance.

(i) Derive expressions for (a) the volume flow rate, (b) the pressure distribution,
(ii) Plot the pressure distribution,
(iii)Calculate maximum pressure developed.

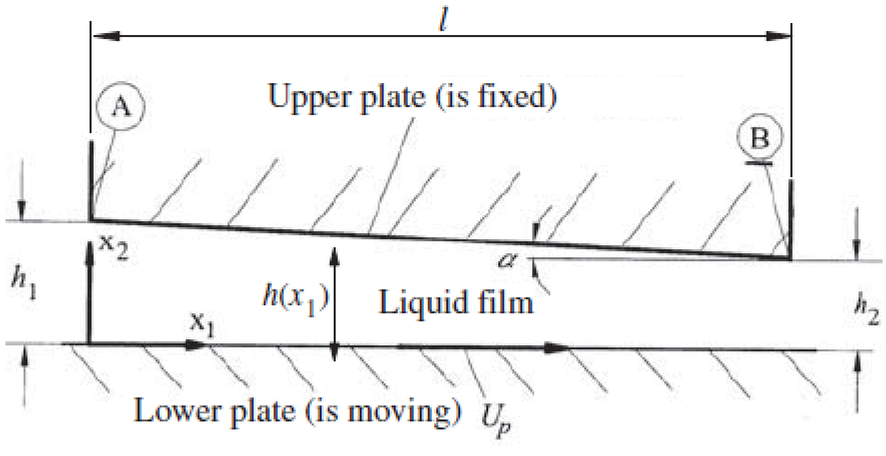

Figure 1. Plane lubrication film flow.
