Department of Mechanical Engineering, BUET.
ME 6125: Mechanics of Viscous Fluid

Assignment-1

(**Due Date: 07 July 2013**, **Sunday**. *Submit hard-copy, at class*)

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1. Derive the y- and z-components of Navier-Stokes equation.

2. The concentration c of fish away from a feeding point in a lake is given by

$$c(x, y) = 1/(x2 + y2).$$

Find the total change of fish concentration detected by an observer riding a boat traveling with speed u =10m/sec straight away from the feeding point.

What is the corresponding change detected by a stationary observer?

3. The two-dimensional velocity field for an incompressible Newtonian fluid is described by the relationship

$$\vec{u} = (12xy^2 - 6x^3)\hat{i} + (18x^2y - 4y^3)\hat{j}$$

Where the velocity has units of m/s when x and y are in meters. Determine the stresses σ_{xx}, σ_{yy}, and τ_{xy} at the point x=0.5 m, y=1.0 m if the pressure at this point is 6 kPa and the fluid is glycerin at 20^0C.

4. The flow of an *incompressible* Newtonian fluid is governed by the *continuity* and the *momentum* equations,

$$\nabla \cdot \mathbf{u} = 0 \,,$$

and

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) \equiv \rho\frac{D\mathbf{u}}{Dt} = -\nabla p + \eta\nabla^2\mathbf{u} + \rho\mathbf{g} \,,$$

where ρ is the density, and **g** is the gravitational acceleration.

Simplify the momentum equation for irrotational flows ($\nabla\times\mathbf{u}=0$).

Note: You may need to invoke both the continuity equation and vector identities to simplify the terms $\mathbf{u}\cdot\nabla\mathbf{u}$ and $\nabla^2\mathbf{u} = \nabla\cdot(\nabla\,\mathbf{u})$.

5. Calculate the velocity-gradient and the vorticity tensors for the following two-dimensional flows and comment on their forms:

(a) *Shear flow*: $u_x = 1 - y$, $u_y = u_z = 0$;

(b) *Extensional flow*: $u_x = ax$, $u_y = -ay$, $u_z = 0$.

Note: The matrix form of the velocity gradient tensor in Cartesian coordinates is

$$\nabla \mathbf{u} = \begin{bmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_y}{\partial x} & \frac{\partial u_z}{\partial x} \\ \frac{\partial u_x}{\partial y} & \frac{\partial u_y}{\partial y} & \frac{\partial u_z}{\partial y} \\ \frac{\partial u_x}{\partial z} & \frac{\partial u_y}{\partial z} & \frac{\partial u_z}{\partial z} \end{bmatrix}$$

Like any tensor $\nabla \mathbf{u}$ can be decomposed into a symmetric, **D**, and an antisymmetric part, **S,**

$$\nabla \mathbf{u} = \mathbf{D} + \mathbf{S},$$

Where, the symmetric tensor $\mathbf{D} = \frac{1}{2}\left[\nabla \mathbf{u} + (\nabla \mathbf{u})^T\right]$ is the *rate of strain* (or *rate of deformation*) *tensor*, and represents the state of the intensity or rate of strain.

and the antisymmetric tensor $\mathbf{S} = \frac{1}{2}\left[\nabla \mathbf{u} - (\nabla \mathbf{u})^T\right]$ is the *vorticity tensor* .

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