Sensor and Actuator Characteristics

- Mechatronic systems use a variety of sensors and actuators to measure and manipulate mechanical, electrical, and thermal systems.
- Sensors have many characteristics that affect their measurement capabilities and their suitability for each application.
- Analog sensors have an output that is continuous over a finite region of inputs.
  – Examples of analog sensors include potentiometers, LVDTs (linear variable differential transformers), load cells, and thermistors.
- Digital sensors have a fixed or countable number of different output values.
  – A common digital sensor often found in mechatronic systems is the incremental encoder.
- An analog sensor output conditioned by an analog-to-digital converter (ADC) has the same digital output characteristics, as seen in Figure 4.1.

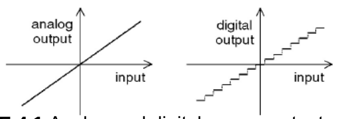


FIGURE 4.1 Analog and digital sensor outputs.

Sensor and Actuator Characteristics

Range

- The range (or span) of a sensor is the difference between the minimum (or most negative) and maximum inputs that will give a valid output.
- Range is typically specified by the manufacturer of the sensor.
- Example: - a common type K thermocouple has a range of 800°C (from −50°C to 750°C).
  – a ten-turn potentiometer would have a range of 3600 degrees.

Resolution

- The resolution of a sensor is the smallest increment of input that can be reliably detected.
- Resolution is also frequently known as the least count of the sensor.
- Resolution of digital sensors is easily determined.
- A 1024 ppr (pulse per revolution) incremental encoder would have a resolution of

$$\frac{1\ \text{revolution}}{1024\ \text{pulses}} \times \frac{360\ \text{degrees}}{1\ \text{revolution}} = 0.3516\frac{\text{degrees}}{\text{pulse}}$$

- The resolution of analog sensors is usually limited only by low-level electrical noise and is often much better than equivalent digital sensors.

Sensor and Actuator Characteristics

Sensitivity

- Sensor sensitivity is defined as the change in output per change in input.
- The sensitivity of digital sensors is closely related to the resolution.
- The sensitivity of an analog sensor is the slope of the output versus input line.
- A sensor exhibiting truly linear behavior has a constant sensitivity over the entire input range.
- Other sensors exhibit nonlinear behavior where the sensitivity either increases or decreases as the input is changed, as shown in Figure 4.2.

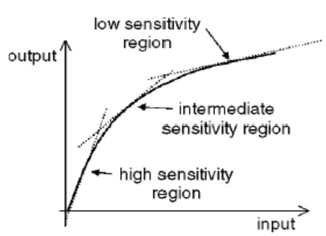


FIGURE 4.2 Sensor sensitivity.

Sensor and Actuator Characteristics

Error

- Error is the difference between a measured value and the true input value.
- Two classifications of errors are
  – bias (or systematic) errors and
  – precision (or random) errors.
- Bias errors are present in all measurements made with a given sensor, and cannot be detected or removed by statistical means.
  – These bias errors can be further subdivided into
  1. calibration errors (a zero or null point error is a common type of bias error created by a nonzero output value when the input is zero),
  2. loading errors (adding the sensor to the measured system changes the system), and
  3. errors due to sensor sensitivity to variables other than the desired one (e.g., temperature effects on strain gages).

Sensor and Actuator Characteristics

Repeatability

- Repeatability (or reproducibility) refers to a sensor's ability to give identical outputs for the same input.
- Precision (or random) errors cause a lack of repeatability. Fortunately, precision errors can be accounted for by averaging several measurements or other operations such as low-pass filtering.
- Electrical noise and hysteresis (described later) both contribute to a loss of repeatability.

Linearity and Accuracy

- The accuracy of a sensor is inversely proportional to error, i.e., a highly accurate sensor produces low errors.
- Many manufacturers specify accuracy in terms of the sensor's linearity.
- A least-squares straight-line fit between all output measurements and their corresponding inputs determines the nominal output of the sensor.
- Linearity is specified
  - as a percentage of full scale (maximum valid input), as shown in Figure 4.3, or
  - as a percentage of the sensor reading, as shown in Figure 4.4.
  - Figures 4.3 and 4.4 show both of these specifications for 10% linearity, which is much larger than most actual sensors.

Sensor and Actuator Characteristics

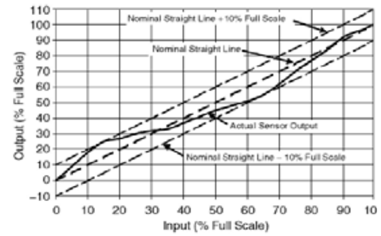


FIGURE 4.3 Linearity specified at full scale.

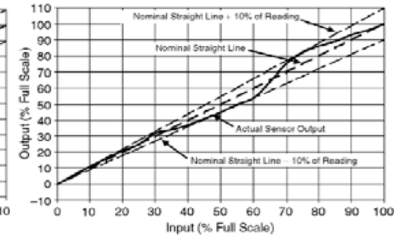


FIGURE 4.4 Linearity specified at reading.

- Accuracy and precision are two terms that are frequently confused.

Accuracy and Precision

- Figure 11.5 shows four sets of histograms for ten measurements of angular velocity of an actuator turning at a constant 100 rad/s.
- First set of data shows a high degree of precision (low standard deviation) and repeatability, but the average accuracy is poor.
- Second set of data shows a low degree of precision (high standard deviation), but the average accuracy is good.
- Third set of data shows both low precision and low accuracy, while
- Fourth set of data shows both high precision, high repeatability, and high accuracy.

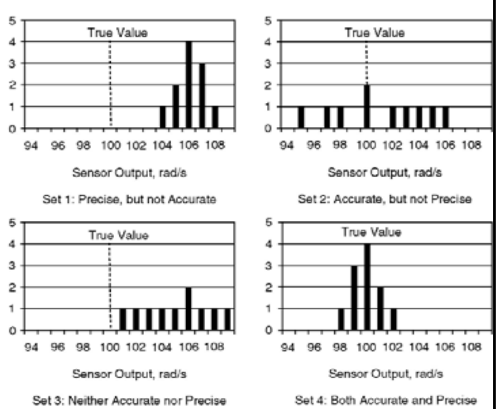


FIGURE 4.5 Examples of accuracy and precision.

Nonlinearities

- Linear systems have the property of superposition.
  - If the response of the system to input A is output A, and the response to input B is output B, then the response to input C (=input A+input B) will be output C (=output A+output B).
- Many real systems will exhibit linear or nearly linear behavior over some range of operation.
- Therefore, linear system analysis is correct, at least over these portions of a system's operating envelope.
- Unfortunately, most real systems have nonlinearities that cause them to operate outside of this linear region, and many common assumptions about system behavior, such as superposition, no longer apply.
- Several nonlinearities commonly found in mechatronic systems include static and coulomb friction, eccentricity, backlash (or hysteresis), saturation, and deadband.

Saturation

- All real actuators have some maximum output capability, regardless of the input.
- This violates the linearity assumption, since at some point the input command can be increased without significantly changing the output; see Figure 11.9.
- This type of nonlinearity must be considered in mechatronic control system design, since maximum velocity and force or torque limitations affect system performance.
- Control systems modeled with linear system theory must be carefully tested or analyzed to determine the impact of saturation on system performance.

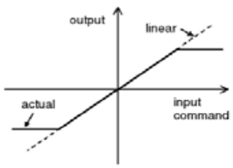


FIGURE 4.6 Saturation.

Deadband

- deadband is typically a region of input close to zero at which the output remains zero.
- Once the input travels outside the deadband, then the output varies with input
- Analog joystick inputs frequently use a small amount of deadband to reduce the effect of noise from human inputs.
- A very small movement of the joystick produces no output, but the joystick acts normally with larger inputs.
- Deadband is also commonly found in household thermostats and other process type controllers. When a room warms and the temperature reaches the setpoint (or desired value) on the thermostat, the output remains off.
- Once room temperature has increased to the setpoint plus half the deadband, then the cooling system output goes to fully on. As the room cools, the output stays fully on until the temperature reaches the setpoint minus half the deadband. At this point the cooling system output goes fully off.

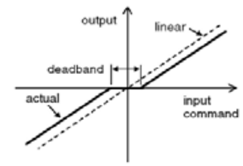


FIGURE 4.7 Deadband.

System Response

- Sensors and actuators respond to inputs that change with time.
- Any system that changes with time is considered a dynamic system.
- Understanding the response of dynamic systems to different types of inputs is important in mechatronic system design.

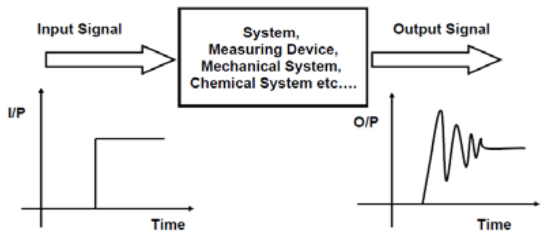


FIGURE 4.8 System response

System Response: input types

- 3 very common inputs are used
  - i. step input,
  - ii. ramp input
  - iii. sinusoidal input.

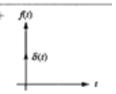
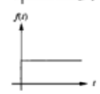
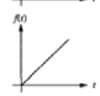
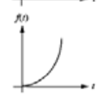
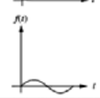
Input	Function	Description	Sketch
Impulse	$\delta(t)$	$\delta(t) = \infty$ for $0- < t < 0+$ $= 0$ elsewhere $\int_{0-}^{0+} \delta(t)dt = 1$	
Step	$u(t)$	$u(t) = 1$ for $t > 0$ $= 0$ for $t < 0$	
Ramp	$tu(t)$	$tu(t) = t$ for $t \geq 0$ $= 0$ elsewhere	
Parabola	$\frac{1}{2}t^2u(t)$	$\frac{1}{2}t^2u(t) = \frac{1}{2}t^2$ for $t \geq 0$ $= 0$ elsewhere	
Sinusoid	$\sin \omega t$		

FIGURE 4.9 System inputs

First-Order System Response

- First-order systems contain two primary elements:
  - an energy storing element and
  - an element which dissipates (or removes) energy.
- Typical first-order systems include resistor–capacitor filters and resistor–inductor networks (e.g., a coil of a stepper motor).
- Thermocouples and thermistors also form first-order systems, due to thermal capacitance and resistance.
- The differential equation describing the time response of a generic first-order system is

$$\frac{dy(t)}{dt} + a\,y(t) = f(t)$$

- Where *y*(*t*) is the dependent output variable (velocity, acceleration, temperature, voltage, etc.), *t* is the independent input variable (time), 1/a is the time constant τ (units of seconds), and *f* (*t*) is the forcing function (or system input).
- The solution to this equation for a step or constant input is given by

$$y(t) = y_\infty + (y_0 - y_\infty)e^{-at} \qquad \text{Eqn (4.1)}$$

First-Order System Response: performance

- Let us examine the significance of parameter *a.* When *t* = *I*, $e^{-at}|_{t=1/a} = e^{-1} = 0.37$

$$y\ (t)|_{t=1/a} = 1 - e^{-at}|_{t=1/a} = 1 - 0.37 = 0.63$$

Time Constant

- the time constant τ can be described as the time for e^{-at} to decay to 37% of its initial value.
- Alternately, the time constant is the time it takes for the step response to rise to 63% of its final value.
- The reciprocal of the time constant has the units (1/seconds), or frequency.
- Thus, we can call the parameter *a* the *exponential frequency*. Since the derivative of e^{-at} is -*a* when *t* = 0, *a* is the initial rate of change of the exponential at *t* = 0.
- Thus, the time constant can be considered a transient response specification for a first order system, since it is related to the speed at which the system responds to a step input.

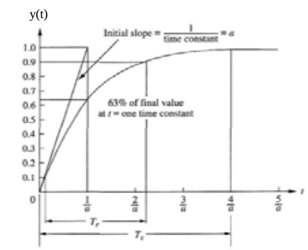


FIGURE 4.10 First-order system response to a unit step

First-Order System Response: performance

Rise Time, T_r

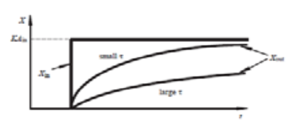
- *Rise time* is defined as the time for the waveform to go from 0.1 to 0.9 of its final value.
- Rise time is found by solving the difference in time at y(t) = 0.9 and y(t) = 0.1. Hence,

$$T_r = \frac{2.31}{a} - \frac{0.11}{a} = \frac{2.2}{a} = 2.2\,\tau$$

Settling Time, T_s

- *Settling time* is defined as the time for the response to reach, and stay within, 2% of its final value.
- Letting y(t) = 0.98 in Eqn. (4.1) and solving for time, *t,* we find the settling time to be

$$T_s = \frac{4}{a} = 4\tau$$



Second-Order System Response

- Second-order systems contain three primary elements: two energy storing elements and an element which dissipates (or removes) energy.
- The two energy storing elements must store different types of energy.
- A typical mechanical second-order system is the spring–mass–damper combination.
- The spring stores potential energy (PE = kx^2), while the mass stores kinetic energy (KE = $1/2mv^2$), where *k* is the spring stiffness (typical units of N/m), *x* is the spring deflection (typical units of m), *m* is the mass (typical units of kg), and *v* is the absolute velocity of the mass (typical units of m/s).
- A common electrical second-order system is the resistor–inductor–capacitor (RLC) network, where the capacitor and inductor store electrical energy in two different forms.
- The generic form of the dynamic equation for an underdamped second-order system is

$$\frac{d^2y(t)}{dt^2} + 2\zeta\omega_n\frac{dy(t)}{dt} + \omega_n^2 y(t) = f(t) \qquad \omega_n = \sqrt{\frac{k}{m}} \qquad B_c = 2\sqrt{mk} \qquad \zeta = \frac{B}{B_c}$$

- where y(t) is the dependent variable (velocity, acceleration, temperature, voltage, etc.), *t* is the independent variable (time), ζ is the damping ratio (a dimensionless quantity), ω_n is the natural frequency (typical units of rad/s), and f(t) is the forcing function (or input). B_c is the critical damping coefficient.

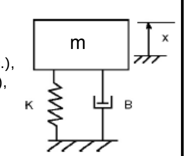


FIGURE 4.11 Spring–mass–damper system.

Second-Order System Response

- The response of an underdamped second-order system to a *unit* step input can be determined as:

$$y(t) = 1 - e^{-\zeta\omega_n t}\left(\cos\omega_n\sqrt{1-\zeta^2}\,t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_n\sqrt{1-\zeta^2}\,t\right)$$

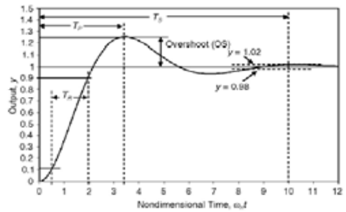


FIGURE 4.12 Second-order system—step response.

Performance parameters

- peak time, T_P: the time required to reach the first (or maximum) peak

$$T_P = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

- percent overshoot, %OS: amount the response exceeds or overshoots the steady-state value

$$\%OS = 100e^{-\left(\zeta\pi/\sqrt{1-\zeta^2}\right)}$$

Second-Order System Response

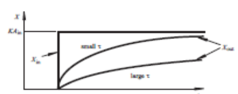
- settling time, T_S: the time when the system response remains within 2% of the steady-state value

$$T_S = \frac{4}{\zeta\omega_n}$$

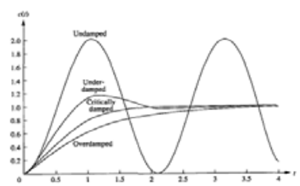
- rise time, T_R: time required for the response to go from 10% to 90% of the steady-state value.
  - Figure 4.12 shows the nondimensional rise time ($\omega_n T_R$) as a function of damping ratio, ζ
  - A frequently used approximation relating these two parameters is

$$\omega_n T_R \approx 2.16\zeta + 0.6 \quad 0.3 \le \zeta\, 0.8$$

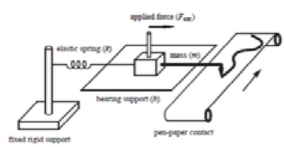
System Response



First-order response



Step responses for second-order system damping cases



Strip chart recorder as an example of a second-order system.

Frequency Response

- The response of any dynamic system to a sinusoidal input is called the frequency response.
- A generic first-order system with a sinusoidal input of amplitude A would have the dynamic equation of

$$\frac{dy(t)}{dt} + \frac{1}{\tau}y(t) = f(t) = A\sin(\omega t)$$

- where ω is the frequency of the sinusoidal input and τ is the first-order time constant. The steady-state solution to this equation is

$$y(t) = AM\sin(\omega t + \Phi)$$

Where $M = 1/\sqrt{(\tau\omega)^2 + 1}$

is the amplitude ratio

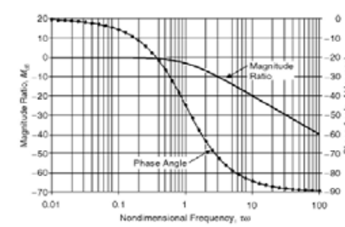
(a dimensionless quantity)

$$\Phi = -\tan^{-1}(\tau\omega)$$

is the phase angle.

Note that the magnitude is frequently plotted in terms of decibels

$$M_{dB} = 20\log_{10}(M).$$



Frequency response for first-order system

Frequency Response

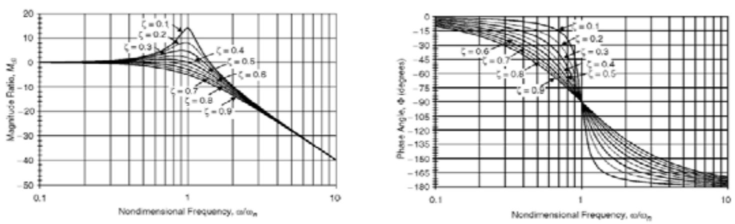
- A generic second-order system with a sinusoidal input of amplitude A and frequency ω would have the dynamic equation of

$$\frac{d^2y(t)}{dt^2} + 2\zeta\omega_n\frac{dy(t)}{dt} + \omega_n^2 y(t) = A\sin(\omega t)$$

Steady state solution $y(t) = \frac{AM}{\omega_n^2}\sin(\omega t + \Phi)$

$$M = \frac{1}{\sqrt{[1-(\omega^2/\omega_n^2)]^2 + [2\zeta(\omega/\omega_n)]^2}}$$

$$\Phi = -\tan^{-1}\left[\frac{2\zeta(\omega/\omega_n)}{1-(\omega^2/\omega_n^2)}\right]$$



Frequency response for second-order system

Frequency Response

- The peak value in the magnitude response, M_p, can be found by taking the derivative of M with respect to ω and setting the result to zero to find (Nise, 1995)

$$M_p = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

- This peak value in M occurs at the frequency ω_P given by

$$\omega_P = \omega_n\sqrt{1-2\zeta^2}$$

- The peak value in an experimentally determined frequency response can be used to estimate both the natural frequency and damping ratio for a second-order system.
- These parameters can then be used to estimate time domain responses such as peak time and percent overshoot.

Poles & Zeros

- The output response of a system is the sum of two responses: the *forced response* and the *natural response.*
- Although many techniques, such as solving a differential equation or taking the inverse Laplace transform, enable us to evaluate this output response, these techniques are laborious and time-consuming.
- The concept of poles and zeros, fundamental to the analysis and design of control systems, simplifies the evaluation of a system's response.

Poles

- The *poles* of a transfer function are

  (1) the values of the Laplace transform variable, *s,* that cause the transfer function to become infinite or

  (2) any roots of the denominator of the transfer function that are common to roots of the numerator.

  - the roots of the characteristic polynomial in the denominator are values of s that make the transfer function infinite, so they are thus poles.

Poles & Zeros

Zeros

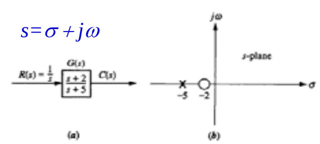
- The *zeros* of a transfer function are

  (1) the values of the Laplace transform variable, *s,* that cause the transfer function to become zero, or

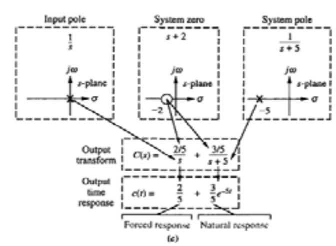
  (2) any roots of the numerator of the transfer function that are common to roots of the denominator.

- the roots of the numerator are values of *s* that make the transfer function zero and are thus zeros.

$s = \sigma + j\omega$



First order system



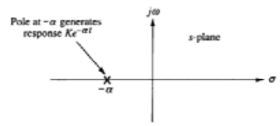
Note: c(t), or, y(t) represents the response.

Poles & Zeros

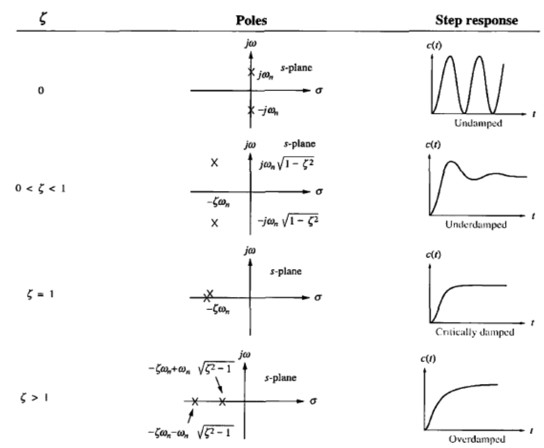
- A pole of the input function generates the form of the *forced response* (that is, the pole at the origin generated a step function at the output).
- A pole of the transfer function generates the form of the *natural response* (that is, the pole at - 5 generated e^{-5t}).
- A pole on the real axis generates an *exponential* response of the form $e^{-\alpha t}$, where $-\alpha$ is the pole location on the real axis. Thus, the farther to the left a pole is on the negative real axis, the faster the exponential transient response will decay to zero (again, the pole at —5 generated e^{-5t}).
- The zeros and poles generate the *amplitudes* for both the forced and natural responses.

Note:

- Each pole of the system transfer function that is on the real axis generates an exponential response that is a component of the natural response.
- The input pole generates the forced response.



Poles & Zeros



%OS for 2nd order system

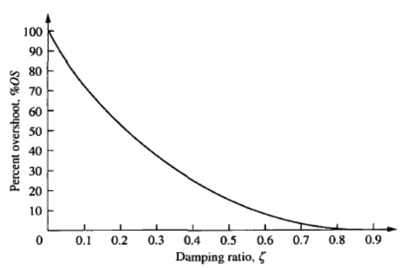
- the percent overshoot is a function only of the damping ratio, ζ.

$$\%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \times 100$$

Which allows one to find %OS for given ζ.

- inverse of the equation allows one to solve for ζ for given %OS which reads

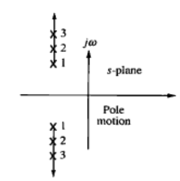
$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}}$$

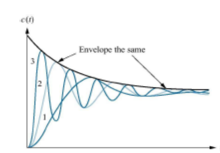


Percent overshoot versus damping ratio

Pole movement: vertical direction

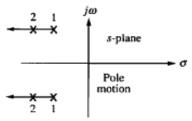
- For step responses, let us move the poles in a vertical direction, keeping the real part the same.
- As the poles move in a vertical direction, the frequency increases, but the envelope remains the same since the real part of the pole is not changing.
- The figure shows a constant exponential envelope, even though the sinusoidal response is changing frequency.
- Since all curves fit under the same exponential decay curve, the settling time is virtually the same for all waveforms.
- Note that as overshoot increases, the rise time decreases.

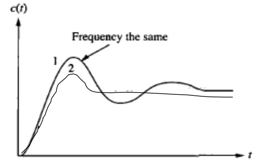




Pole movement: horizontal direction

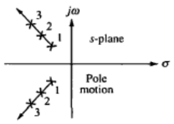
- Let us move the poles to the right or left. Since the imaginary part is now constant, movement of the poles yields the responses shown in the Figure.
- Here the frequency is constant over the range of variation of the real part.
- As the poles move to the left, the response damps out more rapidly, while the frequency remains the same.
- Notice that the peak time is the same for all waveforms because the imaginary part remains the same.





Pole movement: along a radial line

- Moving the poles along a constant radial line yields the responses shown in Figure.
- Here the percent overshoot remains the same.
- Notice also that the responses look exactly alike, except for their speed.
- The farther the poles are from the origin, the more rapid the response.



Note:

- The zeros of a response affect the residue, or amplitude, of a response component but do not affect the nature of the response—exponential, damped sinusoid, and so on.